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MODEL-BASED RECOGNITION OF 3D CURVES FROM ONE VIEW

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Abstract

It is well known there are no geometric invariants of a projection from 3D to 2D. However, given some modeling assumptions about the 3D object, such invariants can be found. The modeling assumptions should be sufficiently strong to enable us to find such invariants, but not stronger than necessary. In this paper we find such modeling assumptions for general 3D curves under affine projection. We show that if we know one of the two affine-invariant curvatures at each point of the curve, we can derive the other one from its image. We can also derive the point correspondence between the curve and the image.

Keywords: object recognition, model based recognition, invariants

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1 Introduction

Almost all the work on invariants so far has been concerned with transformations between spaces of equal dimensionality, e.g. [1, 2]. In the single-view case, invariants were found for the projection of a planar shape onto the image, although the planar shape was embedded in 3D. For real 3D objects, most of the work has involved multiple views with known correspondence, which amounts to a 3D to 3D projection. However, humans have little problem recognizing a 3D object from a single 2D image.

This recognition ability cannot be based on pure geometry, since it has been shown (e.g. [3]) that there are no geometric invariants of a projection from 3D to 2D. Thus, when we only have 2D geometric information, we need to use some modeling assumptions to recover the 3D shape. Such assumptions can be used in a recognition system as follows. We can have a library of 3D objects classified by modeling assumptions. Each class will include all the 3D objects that satisfy a certain assumption. For instance, one class can include all curves with constant torsion, which is the modeling assumption in this case. Another class can include curves which lie on a quadratic or a cubic surface. In the first stage, the recognition process can select the class to which the object belongs, or the modeling assumption. This is beyond the capability of pure geometric reasoning and beyond the scope of this paper. Once a class is selected, an object within it can be identified from the 2D geometric data, if the classification is done appropriately. This is the subject of this paper.

An appropriate classification of the object library has several desirable characteristics. First, the modeling assumptions should be of sufficient strength to enable reconstruction from a 2D image. However, if they are too strong then the classes of objects satisfying them will be too narrow and we will need many different classes. This would complicate the first stage, mentioned above, of choosing the right class. Thus, we need to find *minimal* modeling assumptions that will enable us to reconstruct 3D from 2D.

A second desirable characteristic is that of viewpoint invariance. This has the usual advantage of eliminating the need to find the correct viewpoint. This leads us to express both the classification assumptions and the object descriptions within classes in an invariant way.

Given these characteristics, the second stage of the recognition process can proceed as follows. Invariant descriptors are calculated from the 2D image. These, together with the invariant modeling assumption chosen at the first stage, are used to find the invariant descriptors of the 3D object. These descriptors are matched to descriptors stored in the library, belonging to the same class. Using some indexing scheme, a match can be found without trying every object descriptor stored in the class.

In summary, the invariant modeling assumption and descriptors make it possible to perform recognition regardless of viewpoint and with no need for an exhaustive search of the class.

Earlier work connecting 2D and 3D invariants was done in [4] and [5]. This work did not use modeling assumptions, and therefore a 3D shape could not be recognized uniquely. In [5], five points in 2D were used to find two out of the three invariants that characterize a corresponding 3D quintuple. Thus a whole family of potentially corresponding 3D quintuples was obtained, with one free parameter (apart from an arbitrary affine transformation). Each additional point adds one more free parameter. The need to know the correspondence between the 3D points and their projections is another difficulty. Modeling assumptions have been used in special cases, e.g. in [6, 7]. Here we deal with general curves, and we establish the minimal invariant modeling assumptions needed to find a unique 3D shape and the point correspondence.

2 Point Set Invariants

2.1 General Relations Between 2D and 3D

Here we introduce our method of connecting 3D and 2D invariants by applying it to point sets. We rederive the results in [5] in a much simpler way, using elementary algebra rather than algebraic geometry. In the next section we extend our method to general curves and show how a modeling assumption can be used.

We denote 3D world coordinates by $\mathbf{X}$, and 2D image coordinates by $\mathbf{x}$. We have five points $\mathbf{X}_i$, $i = 1, \ldots, 5$ in 3D space, no four of which are coplanar. They are projected into $\mathbf{x}_i$ in the image. The correspondence is assumed to be known. In a 3D projective or affine

space, five points cannot be linearly independent. We can express the fifth point as a linear combination of the first four:

$$\mathbf{X}_5 = a\mathbf{X}_1 + b\mathbf{X}_2 + c\mathbf{X}_3 + d\mathbf{X}_4 \tag{1}$$

In the projective case the coefficients a, b, c, d are determined only up to a common multiplicative factor so we only have three independent coefficients. In the affine case, the coefficients are constrained by the requirement that the fourth homogeneous coordinate is always 1, again leaving only three independent coefficients. Because the projection from 3D to 2D is linear (in homogeneous coordinates), the same dependence holds in 2D:

$$\mathbf{x}_5 = a\mathbf{x}_1 + b\mathbf{x}_2 + c\mathbf{x}_3 + d\mathbf{x}_4$$

Since determinants are relative invariants of a projective transformation, we look at the determinants formed by these points in both 3D and 2D. Any four of the five points in 3D, expressed in four homogeneous coordinates, can form a determinant M_i. We can give the determinant the same index as the fifth point that was left out. For example,

$$M_1 = |\mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_5|$$

Similarly, in the 2D projection, any three of the five points can form a determinant m_{ij}, with indices equal to those of the points that were left out, e.g.

$$m_{12} = |\mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5|$$

Since the points are not independent, neither are the determinants. Substituting the linear dependence (1) in M_1 above we obtain

$$M_1 = a|\mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_1| + b|\mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_2| + c|\mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_3| + d|\mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_4|$$

As is well known, a determinant with two equal columns vanishes. Also, when columns are interchanged in a determinant, the sign of the determinant is reversed. Therefore we obtain

$$M_1 = a|\mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_1| = -a|\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4| = -aM_5$$

Similarly for the other determinants, with a simplified notation:

$$M_2 = |1,3,4,5| = b|1,3,4,2| = b|1,2,3,4| = bM_5$$

$$M_3 = |1,2,4,5| = c|1,2,4,3| = -c|1,2,3,4| = -cM_5$$

$$M_4 = |1,2,3,5| = d|1,2,3,4| = dM_5$$

The coefficients a, b, c, d can now be expressed as invariants, using the above relations:

$$a = -\frac{M_1}{M_5}, \qquad b = \frac{M_2}{M_5}, \qquad c = -\frac{M_3}{M_5}, \qquad d = \frac{M_4}{M_5} \tag{2}$$

Similar relations hold in the 2D projection:

$$m_{12} = |3,4,5| = a|3,4,1| + b|3,4,2| = a|1,3,4| + b|2,3,4| = am_{25} + bm_{15}$$

$$m_{13} = |2,4,5| = a|2,4,1| + c|2,4,3| = am_{35} - cm_{15}$$

$$m_{14} = |2,3,5| = a|2,3,1| + d|2,3,4| = am_{45} + dm_{15}$$

Other relations are linearly dependent on these. Substituting the coefficients a, b, c, d from eq. (2) in the above relations we obtain three relations between the 3D and the 2D invariants:

$$M_5 m_{12} + M_1 m_{25} - M_2 m_{15} = 0 \tag{3}$$

$$M_5 m_{13} + M_1 m_{35} - M_3 m_{15} = 0 \tag{4}$$

$$M_5 m_{14} + M_1 m_{45} - M_4 m_{15} = 0 \tag{5}$$

These relations are obviously invariant to any affine transformation in both 3D and 2D. A 3D transformation will merely multiply all the M_i by the same constant factor, which drops out of the equations. A 2D affine transformation multiplies all the m_{ij} by the same constant factor, which again drops out. However, in the projective case each point can be independently multiplied by an arbitrary factor λ_i, which does not in general drop out. Thus the above relations are not projectively invariant unless the same coordinates are used for both 3D and 2D, which is not the usual situation in our problem.

2.2 Affine Case

In the affine case, the above relations become linearly dependent so that only two of them are meaningful. To see this, we first note a relationship between the M_i which exists only in the affine case. We can write a determinant involving all 5 points as

$$\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ y_1 & y_2 & y_3 & y_4 & y_5 \\ z_1 & z_2 & z_3 & z_4 & z_5 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

The M_i are minors of this determinant so we can write the above equation as

$$M_1 - M_2 + M_3 - M_4 + M_5 = 0$$

Similar relations can be derived in 2D. We have

$$\begin{vmatrix} x_2 & x_3 & x_4 & x_5 \\ y_2 & y_3 & y_4 & y_5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$$

leading to the relation

$$m_{12} - m_{13} + m_{14} - m_{15} = 0$$

Similarly, from the determinant involving points 1,2,3,4 we obtain the relation

$$m_{15} - m_{25} + m_{35} - m_{45} = 0$$

We now look at the following linear combination of the invariant relations, eqs. (3),(4),(5):

$$(3) - (4) + (5) = M_5(m_{12} - m_{13} + m_{14}) + M_1(m_{25} - m_{35} + m_{45}) + (-M_2 + M_3 - M_4)m_{15} = 0$$

Using the two relations above between the m_{ij} we obtain

$$m_{15}(M_1 - M_2 + M_3 - M_4 + M_5) = 0$$

which is an identity, due the the relation between the M_i above. Thus, only two invariant relations, say (3),(4), are independent. This means that of the three independent invariants

$M_1/M_5, M_2/M_5, M_3/M_5$, only two can be found given m_{ij}. Since all three invariants are needed to characterize the five points in 3D (up to an affine transformation), we cannot find the 3D quintuple uniquely. Similar results were obtained for this case in [5], using Grassmannians, Schubert cycles and wedge products, which are hard to extend to curves.

2.3 Projective Case

The general outline of the derivation is analogous to the previous case and we only summarize it. To obtain invariance we need six points, having two projective invariants. We now have two linear dependencies rather than one:

$$\mathbf{X}_5 = a\mathbf{X}_1 + b\mathbf{X}_2 + c\mathbf{X}_3 + d\mathbf{X}_4$$

$$\mathbf{X}_6 = a'\mathbf{X}_1 + b'\mathbf{X}_2 + c'\mathbf{X}_3 + d'\mathbf{X}_4$$

The determinants M_i are thus again not independent. We can express the dependencies among the M_i with the help of the coefficients $a, \ldots, d'$. The various cross ratios of these determinants are also dependent, and the dependency can be expressed in terms of these coefficients. As we did before, we want to invert the problem and express the coefficients $a, \ldots, d'$ in term of the invariant cross ratios. Of the eight coefficients above, only six are "essential" because of the arbitrary multiplicative factors. However, since we have only two independent cross ratios, only two of the six essential coefficients can be determined, leaving four free parameters.

The linear dependencies above are preserved under projection to 2D, and can be used to find relations between invariant cross ratios of the 2D determinants m_{ij}. These relationships will again contain the coefficients $a, \ldots, d'$, and these can be substituted from the 3D expressions, forming a relation between the 3D and 2D determinants. Unlike the affine case in which all of the coefficients $a, \ldots, d$ could be eliminated, here we will be left with four free parameters. This casts doubt on the usefulness of the exercise in the projective case.

3 Curve Invariants

Here we extend our method to general curves $\mathbf{X}(t)$ in 3D, with their 2D projections $\mathbf{x}(t)$. For the time being we assume we know the correspondence between the 3D and 2D curve

parameters, so we denote them both by t. We will later show how this requirement can be removed using a modeling assumption. This assumption will also eliminate the free parameters resulting from the missing depth information.

We use derivatives in a way analogous to the way we used points before. We deal only with the affine case. The third homogeneous coordinate in this case is always equal to 1, so its derivatives are all 0. Thus we use Cartesian coordinates, and the derivation used before needs to be modified. We can write a linear relation between the first four derivatives in 3D:

$$\mathbf{X}'''' = a\mathbf{X}' + b\mathbf{X}'' + c\mathbf{X}'''$$

We define determinants in analogy to the previous case, but with a lower dimension, e.g.

$$M_1 = |\mathbf{X}'', \mathbf{X}''', \mathbf{X}''''| = |2,3,4|$$

$$m_{12} = |\mathbf{x}''', \mathbf{x}''''| = |3,4|$$

Using the linear dependency above we obtain

$$M_1 = |2,3,4| = a|2,3,1| = a|1,2,3| = aM_4$$

$$M_2 = |1,3,4| = b|1,3,2| = -b|1,2,3| = -bM_4$$

$$M_3 = |1,2,4| = c|1,2,3| = cM_4$$

From this we can recover the invariant coefficients a, b, c as

$$a = \frac{M_1}{M_4}, \qquad b = -\frac{M_2}{M_4}, \qquad c = \frac{M_3}{M_4} \tag{6}$$

Since the affine projection onto 2D is linear, it preserves these coefficients:

$$\mathbf{x}'''' = a\mathbf{x}' + b\mathbf{x}'' + c\mathbf{x}'''$$

Thus the 2D determinants can be written as

$$m_{12} = |3,4| = a|3,1| + b|3,2| = -am_{24} - bm_{14}$$

$$m_{13} = |2,4| = a|2,1| + c|2,3| = -am_{34} + cm_{14}$$

$$m_{23} = |1,4| = b|1,2| + c|1,3| = bm_{34} + cm_{24}$$

These relations are not independent. To see this, we note that in 2D we have a linear relation

$$\mathrm{x}''' = \alpha \mathrm{x}' + \beta \mathrm{x}'' \tag{7}$$

with some coefficients α, β. Substituting this in the determinants in the equations above, it is easy to see that the first equation is the sum of the remaining two.

We now substitute the invariants a, b, c from eq. (6) in the last two relations between the m_{ij} above to obtain the relation between the 2D and 3D invariants:

$$M_4 m_{13} + M_1 m_{34} - M_3 m_{14} = 0 \tag{8}$$

$$M_4 m_{23} + M_2 m_{34} - M_3 m_{24} = 0 \tag{9}$$

These equation are simplified when we use a parameter which is affine invariant in 2D. Such a parameter can be defined so that [8]

$$m_{34} = |\mathrm{x}', \mathrm{x}''| = 1$$

This parameter can be obtained from any arbitrary parameter s by

$$t = \int |\mathrm{x}_s, \mathrm{x}_{ss}|^{1/3} ds$$

All derivatives denoted by a prime are with respect to this 2D invariant parameter t. Since m_{34} is only a relative invariant, t is also a relative invariant, i.e. it is an affine invariant only up to a constant factor. However, it is easy to show that this factor drops out of the equations. In this system other 2D determinants are simplified too. Differentiating $m_{34} = 0$ we obtain

$$m_{24} = |\mathrm{x}', \mathrm{x}'''| = |\mathrm{x}', \mathrm{x}''|' = 0$$

and differentiating the above relation

$$|\mathrm{x}', \mathrm{x}'''|' = |\mathrm{x}'', \mathrm{x}'''| + |\mathrm{x}', \mathrm{x}''''| = 0$$

we obtain

$$m_{14} + m_{23} = 0$$

With these expressions, the 2D to 3D relations (8), (9) simplify to

$$M_4 m_{13} - M_3 m_{14} + M_1 = 0 \tag{10}$$

$$M_4 m_{14} - M_2 = 0 \tag{11}$$

As in 2D, we define an invariant parameter τ in 3D, which is different from the 2D invariant parameter t:

$$\tau = \int |\mathbf{X}', \mathbf{X}'', \mathbf{X}'''|^{1/6} dt$$

τ is invariant only up to a constant factor, which drops out of the equations. In this system some of the determinants M_i simplify:

$$\bar{M}_4 = |\mathbf{X}^*, \mathbf{X}^{**}, \mathbf{X}^{***}| = 1$$

$$\bar{M}_3 = |\mathbf{X}^*, \mathbf{X}^{**}, \mathbf{X}^{****}| = |\mathbf{X}^*, \mathbf{X}^{**}, \mathbf{X}^{***}|^* = 0$$

with the bars denoting the dependence of $\bar{M}_i(\tau)$ on τ, and the stars denoting differentiation with respect to τ. The remaining two $\bar{M}_i$ are relative affine invariants characterizing the curve. They are similar to the "affine curvatures" k_1, k_2 defined in [8]:

$$\bar{M}_1 = k_1, \qquad \bar{M}_2 = -k_2$$

Given these two 3D affine curvatures, the original curve can be reconstructed up to a 3D affine transformation [8].

We would like to build a library of 3D shapes characterized by invariant descriptors. The above two invariants with an invariant parametrization are a good choice for such descriptors because they are sufficient to characterize the curve. Thus we want to relate them to our 2D invariants m_{ij}. However, these quantities m_{ij}, as well as the relations (10), (11), are functions of the non-3D-invariant parameter t. Thus, our task is now to transform from the invariant $\bar{M}_i(\tau)$ to the non-invariant $M_i(t)$.

We define

$$j = \frac{d\tau}{dt}(t)$$

The coordinates transform as follows, with $\mathbf{X}' = \frac{d\mathbf{X}}{dt}(t)$, $\mathbf{X}^* = \frac{d\mathbf{X}}{d\tau}(\tau)$:

$$\mathbf{X}' = \mathbf{X}^* j$$

$$\mathbf{X}'' = \mathbf{X}^{**} j^2 + \mathbf{X}^* j'$$

$$\mathbf{X}''' = \mathbf{X}^{***} j^3 + 3\mathbf{X}^{**} j j' + \mathbf{X}^* j''$$

$$\mathbf{X}'''' = \mathbf{X}^{****} j^4 + 6\mathbf{X}^{***} j^2 j' + \mathbf{X}^{**}(4j'' j + 3j'2) + \mathbf{X}^* j'''$$

We use these relations to express $M_i(t)$ in terms of $\bar{M}_i(\tau)$. This is done as follows. First, the column with the lowest order derivative in $M_i(t)$ is replaced by the corresponding expression involving τ from the above relations. This yields several determinants, some of which vanish due to having equal columns. In the surviving determinants we proceed in the same way to the higher order derivatives. We will show the explicit calculation only in the simplest cases. In the simplified notation below, we denote $\mathbf{X}'(t)$ inside a determinant by 1, $\mathbf{X}^*(\tau)$ by $\bar{1}$, etc.

$$M_4 = |1,2,3| = j|\bar{1},2,3| = i^3|\bar{1},\bar{2},3| = j^6|\bar{1},\bar{2},\bar{3}| = j^6\bar{M}_4 = j^6$$

$$M_3 = |1,2,4| = j|\bar{1},2,4| = j^3|\bar{1},\bar{2},4| = 6j^5j'|\bar{1},\bar{2},\bar{3}| + j^7|\bar{1},\bar{2},\bar{4}| = 6j^5j'$$

where we have used the facts that $|\bar{1},\bar{2},\bar{3}| = 1$ (an ordinary 1), and $|\bar{1},\bar{2},\bar{4}| = 0$. In a similar fashion we obtain (after a longer calculation)

$$M_1 = 15j^3j'^3 - 10j^4j'j'' + j^5j''' + j^7j'\bar{M}_2 + j^9\bar{M}_1$$

$$M_2 = 15j^4j'^2 - 4j^5j'' + j^8\bar{M}_2$$

Substituting this in the 2D to 3D relations (10),(11) we obtain

$$m_{13}j^3 - 6m_{14}j^2j' + 15j'^3 - 10jj'j'' + j^2j''' + j^4j'\bar{M}_2 + j^6\bar{M}_1 = 0$$

$$m_{14}j^2 - 15j'^2 + 4jj'' - j^4\bar{M}_2 = 0$$

These equations can be simplified by eliminating $\bar{M}_2$ from the first equation and substituting $u = 1/j$:

$$m_{14}u^2 - 7u'^2 - 4uu'' = \bar{M}_2 \tag{12}$$

$$m_{13}u^3 + 5m_{14}u^2u' + 6u'^3 - u^2u''' = -\bar{M}_1 \tag{13}$$

We have here two equations in three unknowns: the correspondence function $u(t)$, and the two affine invariant curvatures $\bar{M}_1, \bar{M}_2$. To solve it we need to know one of the unknowns, or some relation between the unknowns.

Here is where a modeling assumption is needed to supply the missing information. One example of such an assumption is that one of the $\bar{M}_i$ is a constant, or some other simple

function. Another example is assuming that the curve lies on a quadric or a cubic surface. This provides a relation between the curvatures $\bar{M_1}, \bar{M_2}$, namely a third equation. Given such modeling information, we can solve the system of equations and thus recover the 3D curve from the 2D image. This includes the recovery of the correspondence information, through $u(t)$. In other words, we do not need to know the correspondence of curve points beforehand.

If we assume that one of the curvatures is constant, it stays constant regardless of the parametrization, and we obtain a differential equation for $u(t)$ which can be solved either analytically (in simple cases) or numerically. If a curvature is not constant, it can be written in the above equation as

$$\bar{M_i}(\tau(t)) = M_i(\int 1/u(t)dt)$$

so we have an integro-differential equation. We still have up to three free parameters in the solution for u, but they are the same parameters for the whole curve. Without the modeling assumption we would have free parameters at each point. The search for these parameters can be greatly simplified by the fact that t is locally a quasi-invariant of the projection, when the affine curvatures are not too large. That is, we can calculate t up to a slowly varying scale factor. Thus, we can calculate a good approximation to dt around each 3D curve point, by projecting the curve onto the local osculating plane from the direction of the affine bi-normal. From this, a local approximation to $u(\tau) = dt/d\tau$ can be found and stored in the library, and later used to eliminate most of the search space.

In an object recognition system for curves, we can classify the curves according to, e.g., the kinds of surfaces they are assumed to lie on. Within such a class, the curvatures of different curves can be indexed in some way. Now, given a 2D image of a curve known to lie on one of the surfaces used in the library, we can recover its curvatures from the above equations, and then match it with the indexed library. No complex search or point correspondence is needed.

4 Conclusion

We have used a model-based approach to assist in the recognition of 3D objects from single images. We have shown how a modeling assumption can be used to substitute for the missing depth information. We have dealt with general curves under affine projection, and have obtained a system of two differential equations in three unknown functions. Two unknowns represent the two 3D affine curvatures characterizing the curve, and the third represents the correspondence information. A modeling assumption provides a third equation, closing the system. For instance, we can assume that the curve lies on a quadric surface. This provides a relation between the curvatures, and we can recover both. So, if a modeling assumption is known, a 2D image can be used to recognize a 3D object, eliminating the need for extensive search or point correspondence.

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